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Balancing of an Unbalanced Jeffcott Rotor Using Artificial Neural Network to Predict the Correction Mass and Phase Angle

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Abstract

In the modern era, technology's undeniable impact on various industries is particularly prominent in rotating machinery applications, including engine rotors and industrial turbomachinery. A common challenge in this domain is rotor imbalance, leading to detrimental consequences such as excessive vibrations, bearing wear, and machinery breakdowns. This study aims to investigate Jeffcott rotor dynamics through precise MATLAB modeling, intentionally introducing mass imbalance to simulate real-world scenarios accurately. A Vector-Balancing technique is employed to address this imbalance, and an Artificial Neural Network (ANN) is trained to proactively mitigate rotor imbalances, drawing upon the results obtained from the modeling process. This enables the ANN to effectively address rotor imbalances, enhancing the reliability and performance of rotating machinery applications. To validate the effectiveness of this ANN-based approach in addressing rotor imbalance issues across various rotating machinery applications, physical validation is carried out using a physically constructed rotor model. The VIBXPERT device balances the physical model, confirming the reliability and efficacy of this approach. This research underscores the practical applicability of ANN-based strategies in effectively addressing rotor imbalance issues, ensuring the longevity and reliability of diverse rotating machinery applications.

Keywords: Rigid Balancing; Artificial Neural Network; Jeffcott Rotor; Rotating Machinery

1. Introduction

In today's industries, rotating machinery plays a pivotal role, but it often faces challenging conditions such as heavy loads, high temperatures, and rapid speeds, making it prone to breakdowns [1]. These breakdowns can lead to reduced efficiency, significant economic losses, and even safety hazards, necessitating immediate shutdowns in some cases [2].

One common issue causing breakdowns in rotating machinery is dynamic imbalance, where the center of mass of a rotating component is not aligned with its axis due to various factors like irregular casting, lack of concentricity, or foreign objects [3]. This imbalance generates centrifugal forces during operation, resulting in undesirable vibrations, increased wear, decreased performance, and potentially catastrophic failures [4]. To address dynamic imbalance, specific balancing techniques are used for rigid and flexible rotors, such as the Vector-Balancing Method for rigid rotors and the Influence Coefficient Method for flexible ones [5]. Renowned researchers like RAO, VANCE, CHILDS, and Parkinson have extensively discussed these techniques [6][7][8][9].

In recent times, Artificial Intelligence (AI) has made significant strides in mechanical systems and balancing techniques. AI, including Artificial Neural Networks, has been applied to improve the efficiency and accuracy of balancing processes in rotating machinery [10]. ANN can predict correction masses based on unbalanced responses, offering a faster and more automated solution to balancing [11]. Researchers have successfully employed ANN for diagnostics, instability control, fault prediction, and fault diagnosis in rotating machines [12]. However, more studies need to be utilizing AI techniques for predicting correction masses and phase angles in balancing systems.

This study focuses on balancing a Jeffcott rigid rotor supported by two ball bearings, utilizing the Vector Method and artificial neural networks for enhanced balancing. The method involves initial measurements of vibration signal amplitude and phase angle, then introducing a trial mass to the rotor and re-measuring these parameters. By comparing the initial and post-trial mass measurements, the correction mass value is determined based on phase differences [13]. An artificial neural network is trained using unbalanced responses and correction masses obtained through the Vector Method. This innovative approach can predict correction masses when unbalanced responses are input, significantly improving the balancing process. The method is validated by comparing predictions to physical experiments on a fabricated rotor. It demonstrates its reliability in addressing rotor imbalance and its potential to enhance balancing processes for various rotating systems.

2. Rotor-Bearing System Modelling

The widely used model for analyzing unbalanced rotating bodies, named after its creator, H. H. Jeffcott, in 1919 [14], simplifies an unbalanced rotor into a linear system. It presents a linear model consisting of a substantial unbalanced disc in the center of an unsubstantial elastic shaft bedded in two rigid bearings. In its complete form, this model is described by four second-order differential equations representing four degrees of freedom. However, when simplifying the disc as a mass point, the rotor effectively exhibits only two degrees of freedom, allowing movement solely along the radial direction along horizontal and vertical axes. As the rotor rotates, the center of gravity follows a well-known trajectory known as the orbit [15]. Fig. 1 illustrates a simplified dynamic representation of this rotor model, considering the flexural rigidity and damping of the bearings, which can be thought of as springs and dampers that move in sync with the rotor. Consequently, the rotor is connected to the ground via linear springs and dampers, and its motion in the \bar{y} and \bar{z} directions is influenced by the time-varying radial components of the rotating force vector.

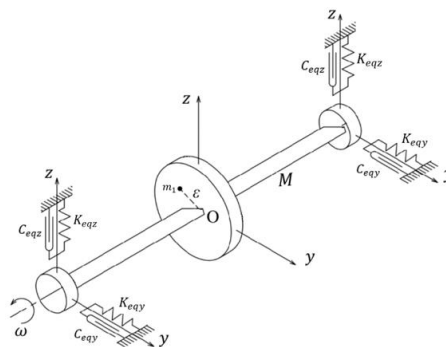


Figure 1. Model of the Jeffcott-like rotor [16].

The equations of motion are derived using the Lagrange operator by correctly identifying the forces acting on the system with respect to the generalized coordinates [17].

$$(m_1 + M)\ddot{y} + K_{eqy}y + C_{eqy}\dot{y} - m_1\varepsilon\omega(t)^2 \cos\left(\int_0^t \omega(t)dt + \varphi_1\right) - m_1\varepsilon\dot{\omega}(t) \sin\left(\int_0^t \omega(t)dt + \varphi_1\right) \quad (1)$$

$$(m_1 + M)\ddot{z} + K_{eqz}z + C_{eqz}\dot{z} - m_1\varepsilon\omega(t)^2 \sin\left(\int_0^t \omega(t)dt + \varphi_1\right) + m_1\varepsilon\dot{\omega}(t) \cos\left(\int_0^t \omega(t)dt + \varphi_1\right) \quad (2)$$

Where m_1 is the unbalance mass, M is the rotor mass, K_{eqy} and K_{eqz} are the equivalent stiffness coefficient along the \bar{y} and \bar{z} direction, and C_{eqy} and C_{eqz} are the equivalent damping coefficient along the \bar{y} and \bar{z} direction, ε is the radial displacement of the unbalance mass, ω is the angular velocity of the rotor, and φ_1 is the initial angular position of the unbalance mass.

3. Experimental Setup and Procedure

The experimental setup, depicted in Fig. 2, was designed to analyze an unbalanced Jeffcott rotor. It comprises a three-phase induction motor (Motogen 63-2B) connected to an aluminum shaft with two plexiglass disks for off-center mass placement, and it utilizes SKF6006 bearings for shaft support. An inverter (Teco-S310) was employed to convert municipal electricity to three-phase power to address the unavailability of industrial three-phase power. Additionally, a complete SolidWorks setup model was created to ensure precise design and alignment.

The rotor, tested at 750 rpm (12.5 Hz) with unbalanced mass placed on the outer diameter of the disks ($\varepsilon = 150$ mm), was monitored using a split CLD accelerometer sensor magnetically attached to the motor-side bearing in the vertical direction. Also, a laser-triggered optical was employed for vibration measurements and RPM monitoring. Ultimately, with the assistance of the VIBXPERT II device, correction mass and phase angle were determined, ensuring rotor balance and minimizing unwanted vibrations.

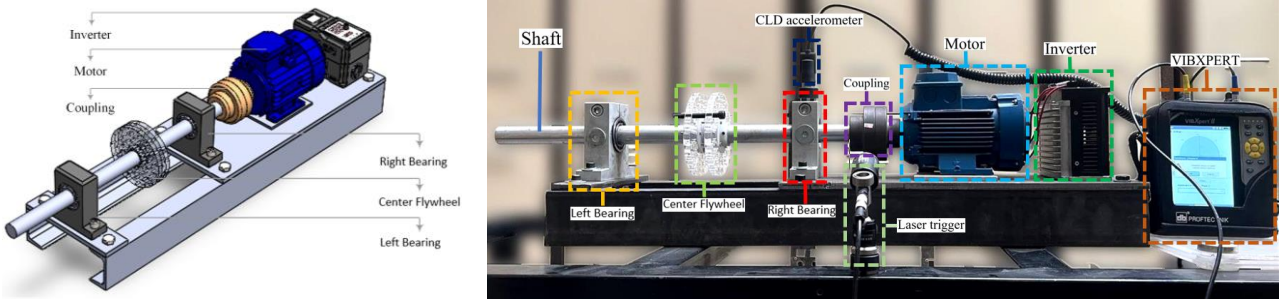


Figure 2. SolidWorks prototype and experimental setup.

4. Vector-Balancing Method and Solution Approach

The balancing process comprises several steps, which encompass measuring the initial vibrations, introducing a trial mass, gauging the secondary vibrations, and ultimately applying the correction mass. These individual steps will be examined in greater detail in the subsequent sections.

4.1 Initial Conditions and Input Parameters

In this study, the rotor bearing system was modeled using MATLAB 2020b software with the help of information and characteristics of the constructed physical model. The rotor parameters employed in the modeling process are described in Table. 1. The length and diameter are such that the rotor can be considered a rigid structure, and the material employed simulates a common one in the industry.

Table 1. Rotor parameters.

Rotor Parameters	
Length [m]	0.6
Diameter [m]	0.03
Mass Density [kg m^{-3}]	2830

In this system, the dynamic coefficients of damping and stiffness used in the MATLAB modeling are presented in Table. 2. These coefficients were determined through the Half-Power Bandwidth method and by identifying the resonance frequency at each unbalanced state [18].

Table 2. Dynamic coefficients.

System-Equivalent Dynamic Coefficients	
Stiffness Coefficients [kN m^{-1}]	
K_{eqy}	K_{eqz}
39.525	37.002
Damping Ratio	
ζ_{eqy}	ξ_{eqz}
0.0135	0.0088

In this modeling, the exponential change in rotational velocity of the shaft from a stationary state approximates the dynamic model to the actual state, as illustrated in Fig. 3. This approach ensures that the model closely represents the real-world behavior of the system.

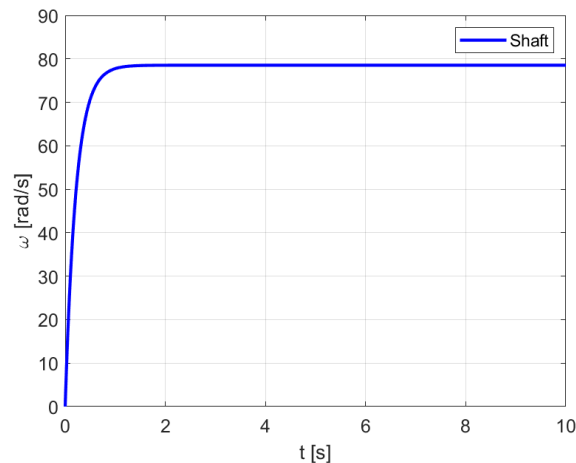


Figure 3. Rotor and shaft angular velocity.

In the MATLAB software, the ode45 command was employed to solve the differential equations governing the vibrations described by Eqs. (1) and (2). The resulting outputs, which illustrate velocity versus time in the \vec{y} and \vec{z} directions, are presented in Fig. 4. This simulation provides valuable insights into the system's dynamic response.

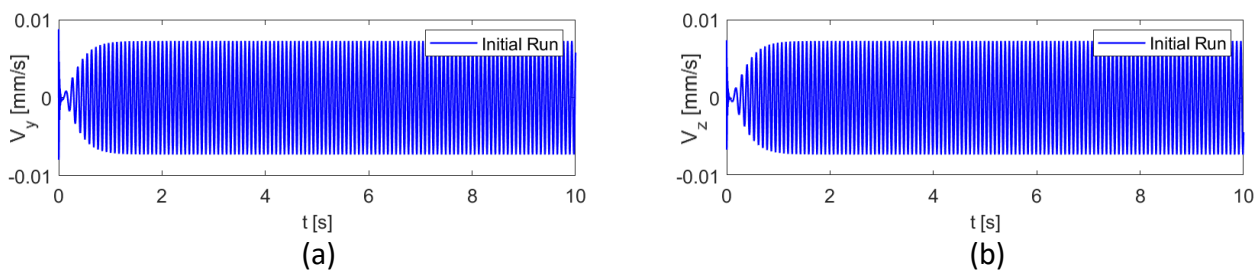


Figure 4. Initial velocity in (a) \vec{y} direction, (b) \vec{z} direction.

4.2 Trial Mass

The initial step in the balancing process involves adding a trial mass at a radial distance of ε from the disk's center and an angle θ to the system. Similar to Eqs. (1) and (2), the equations for the new system can be derived. Upon adding the trial mass, the system's vibration response changes, as depicted in Fig. 5. The primary goal is to determine the phase difference between the trial and unbalanced mass, equivalent to the phase difference between the old and new vibration signals. Based on this phase difference, the position of the unbalanced mass, which is essentially the unknown in the problem, can be determined. In practical applications, this is achieved using a tachometer, which is simulated in this code by generating a pulse signal. Consequently, the phase difference between the two signals is calculated concerning these pulses.

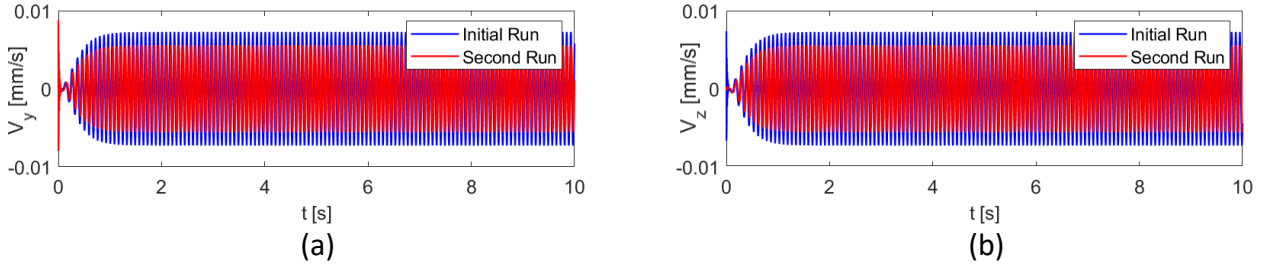


Figure 5. Initial and secondary velocities in (a) \vec{y} direction, (b) \vec{z} direction.

4.3 Correction Mass

After pinpointing the exact location of the equivalent unbalanced mass, the subsequent step involves computing the correction mass's magnitude. This calculation relies on the trial mass value and the amplitudes of the initial and secondary vibrations, and it can be accomplished using the following Eq. (3) and Fig. 6.

$$CW = (TW) \times \frac{|\vec{O}|}{|\vec{T}|} \tag{3}$$

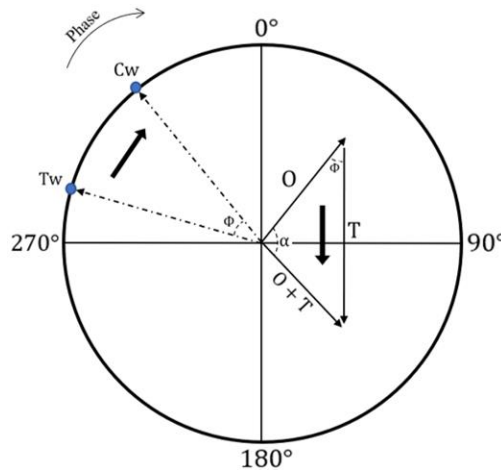


Figure 6. Vector-Balancing Method.

Once the value of the correction mass is determined, it should be positioned precisely opposite the unbalanced mass, with a phase difference of 180 degrees. After adding the correction mass, the system's vibration can be observed, as depicted in Fig. 7. It is evident that the vibration level is significantly reduced, and due to the accuracy of this performance, the results of vector-based balancing can be used for training the neural network model.

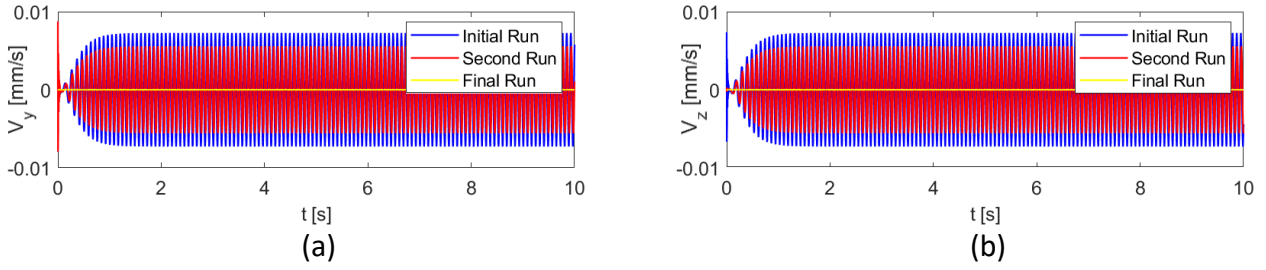


Figure 7. Initial, secondary, and final velocities in (a) \bar{y} direction, (b) \bar{z} direction.

5. Artificial Neural Network Balancing Methodology

The novelty of the proposed balancing methodology in this study lies in its utilization of Artificial Neural Networks to balance rotors based on Vector-Balancing Method results. Neural networks are intricate, self-adaptive networks composed of interconnected elements known as neurons. While neurons perform fundamental computations, their interactions empower the ANN to learn from input data and corresponding outputs [19].

This research harnesses a Multilayer Perceptron Network, offering greater computational power than a single-layer neural network. It comprises an input layer, eight hidden layers, and an output layer. In the context of the balancing process, artificial intelligence and machine learning algorithms, such as neural networks, prove highly effective. Vibration measurements and system characteristics serve as inputs to a neural network, which, through machine learning techniques, can discern patterns in system vibrations and automatically compute the optimal balancing mass [20].

Training the neural network with the unbalanced responses obtained from mathematical modeling of the rotor bearing system and correction masses derived from the Vector-Balancing technique enables the network to discover hidden patterns within the data and estimate the balancing mass value. Once trained, the neural network can be integrated into the balancing procedure. When inputting new data into the network, it computes the correction mass value as its output. This calculated balancing mass is then applied to the appropriate location within the system, thereby finalizing the balancing process. Using artificial intelligence and machine learning algorithms in the balancing process offers several advantages, including improved accuracy, faster processing times, and increased efficiency in system balancing. Furthermore, this approach enhances adaptability to system changes and diminishes the necessity for manual adjustments.

5.1 Network Design

Building a neural network model begins with having an adequate dataset for the training process. In this study, a MATLAB-based modeling approach utilized a Vector-Balancing method. It was employed within a loop to randomly generate the unbalanced mass and its position at each stage while keeping the trial mass and its position constant. This process was iterated to generate 1000 data points, which were subsequently saved. For this operation, the Neural Net Fitting module in MATLAB was utilized. The first step in using this module involves specifying the inputs and targets. The primary and secondary vibration signal levels and their phase angles were designated as inputs in this case. Simultaneously, the correction mass, its phase angle, the final vibration signal level, and its phase angle were set as targets. The dataset was then divided into subsets, with 5% for testing, 20% for validation, and 75% for training.

5.2 Artificial Neural Network Parameters

In the learning process of this study, the Levenberg-Marquardt algorithm was employed as the optimization method. This algorithm, detailed in Haykin's book, is a widely used optimization technique for training artificial neural networks [21]. The Levenberg-Marquardt algorithm adapts the Gauss-Newton algorithm and combines the strengths of both the Steepest Descent and Gauss-Newton methods. Its primary objective is to minimize the error between the network's output and the desired

output by adjusting the synaptic weights [22]. Table. 3 describes the critical parameters used in training the artificial neural network during this learning stage. These parameters hold a pivotal role in the training process and significantly impact the network's convergence and accuracy.

Table 3. Artificial neural network parameters.

Artificial Neural Network Parameters	
Transfer function	Sigmoid function
Rate of learning	0.05
Increase factor of learning	1.05
Performance goal	0.001
Momentum	0.075
Interactions	3,000

6. Results

To evaluate the performance of the neural network model developed in this study, a dataset consisting of 45 random data points within the input range of the neural network model was generated. Subsequently, the outputs for the correction mass and phase angle were compared between the modeling performed in MATLAB and the neural network model, as depicted in Fig. 8.

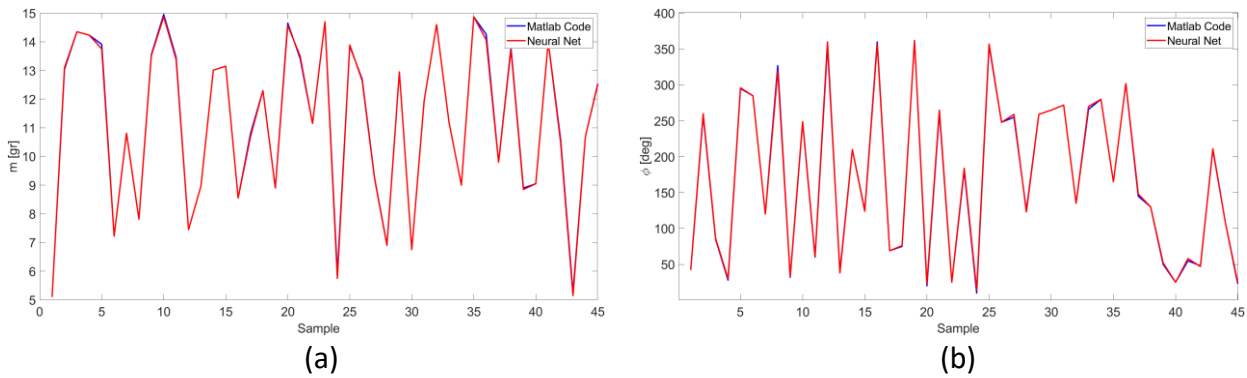


Figure 8. Comparison of (a) correction mass, (b) phase angle.

To evaluate the neural network model's performance, a validation procedure was conducted using real-world conditions, wherein a physical model with an unbalanced mass of 15 grams and a phase angle of 130 degrees, rotating at a speed of 750 rpm, was balanced with the assistance of VIBXPRT. The results of this physical balancing experiment have been presented in Table. 4 and were compared to the predictions from the neural network model.

Table 4. Comparison of experimental and neural network model results.

	Experimental Results	ANN Model Results	Relative Error [%]
Correction Mass	15.53 [gr]	14.98 [gr]	3.54
Phase Angle	-51.69°	-49.99°	3.28

7. Conclusion

In conclusion, this study aimed to improve the accuracy of outcome predictions in the field of rotor balancing by integrating advanced modeling techniques and machine learning approaches. A comprehensive model of the Jeffcott rotor was developed using MATLAB software, which incorporated dynamic force coefficients and employed vectorial balancing techniques. This model was the

foundation for training an ANN to predict the necessary correction data required for effective system balancing, including masses and angles. The ANN, configured with eight hidden layers and employing the Levenberg-Marquardt training algorithm, consistently demonstrated exceptional performance, maintaining an error rate well below the 0.1% threshold. The validation process, which involved using a physical model to simulate real-world conditions and the VIBXPRT for balancing, further verified the network's outstanding accuracy.

These results highlight the promising potential of the ANN-based approach for practical rotor balancing applications. By bridging the gap between advanced modeling and machine learning, this research contributes significantly to advancing rotor balancing techniques. It underscores the critical role of incorporating cutting-edge technologies to improve outcomes in this field. As rotor systems play a vital role in various industrial applications, adopting such innovative methodologies promises to enhance their efficiency and performance.

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